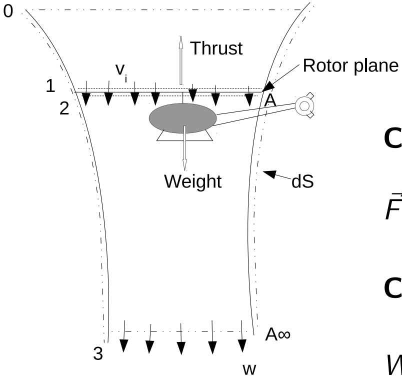
Momentum Theory - 1



Conservation of Mass :

$$\iint_S \rho \vec{V} \cdot \vec{dS} = 0$$

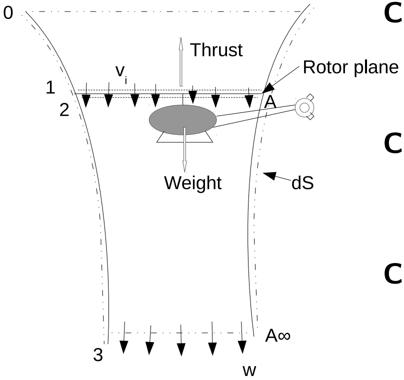
Conservation of Momentum:

$$\vec{F} = \iint_S p\vec{dS} + \iint (\rho \vec{V} \cdot \vec{dS})\vec{V}$$

Conservation of Energy:

$$W = \iint_S \frac{1}{2}(\rho \vec{V}\vec{dS})|\vec{V}|^2 + \iint_S p(\vec{V} \cdot \vec{ds})$$

Momentum Theory - 2



Conservation of Mass:

$$\dot{m} = \rho A v_i = \rho A_3 w$$

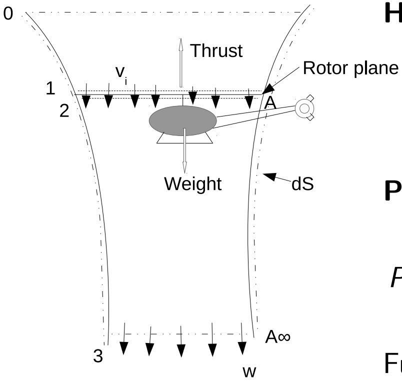
Conservation of Momentum:

$$T = \dot{m} w$$

Conservation of Energy:

$$T v_i = \frac{1}{2} \dot{m} w^2$$

Momentum Theory - 3



$$w = 2v_i$$

Hover induced velocity:

$$v_i = \sqrt{\frac{T}{2\rho A}} = v_h$$

Power

$$P = Tv_i = T\sqrt{\frac{T}{2\rho A}} = \frac{T^{\frac{3}{2}}}{\sqrt{2\rho A}}$$

Fundamental relations between thrust, disk area, power and induced velocity.

Non-Dimensional Quantities

Thrust coefficient:

$$C_T = \frac{T}{\rho A (\Omega R)^2}$$

For helicopters:
$0.004 \leq C_T \leq 0.014$
Power coefficient:

$$C_P = \frac{P}{\rho A (\Omega R)^3}$$

Torque coefficient:

$$C_Q = \frac{Q}{\rho A (\Omega R)^2 R} = C_P$$

Induced inflow:

$$\lambda_i = \lambda_h = \sqrt{\frac{C_T}{2}}$$

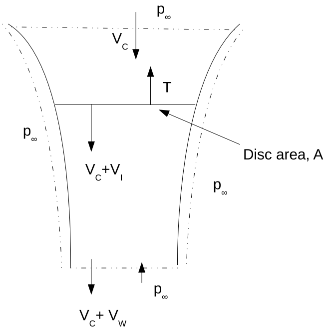
Power coefficient:

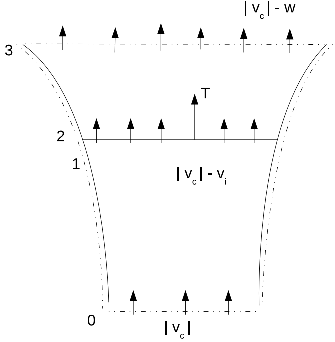
$$C_P = \frac{T^{\frac{3}{2}}}{\rho A (\Omega R)^3 \sqrt{2 \rho A}}$$

$$C_P = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} = C_T \cdot \lambda_h$$

Disk Loading: $DL = \frac{T}{A}$
Power Loading: $PL = \frac{T}{P}$

Momentum Theory - Climbing and Descending Rotor





$$\frac{v_i}{v_h} = \sqrt{(\frac{v_c}{2v_h})^2 + 1} - \frac{v_c}{2v_h}$$

$$\frac{v_i}{v_h} = -(\frac{v_c}{2v_h}) - \sqrt{(\frac{v_c}{2v_h})^2 - 1}$$

valid for $\frac{v_c}{v_h} \leq -2$

Inflow For General Descending Flight

$-2 \leq \frac{v_c}{v_h} \leq 0$ no simple theory
From curve fitting to experimental data, two empirical relations have been obtained:

$$\frac{v_i}{v_h} = \begin{cases} k - \frac{3v_c}{4v_h}, & \text{for } \frac{-8k}{4k+1} \leq \frac{v_c}{v_h} \leq 0 \\ k[7 + \frac{3v_c}{v_h}], & \text{for} -2 \leq \frac{v_c}{v_h} < \frac{-8k}{4k+1} \end{cases}$$

Where, k = measured induced power factor (typically 1.15)
continuous quartic fit:

$$\frac{v_i}{v_h} = k + k_1 \frac{v_c}{v_h} + k_2 (\frac{v_c}{v_h})^2 + k_3 (\frac{v_c}{v_h})^3 + k_4 (\frac{v_c}{v_h})^4$$

$k_1 = -1.125, k_2 = -1.372, k_3 = -1.718, k_4 = -.655$ is valid for $-2 \leq \frac{v_c}{v_i} \leq 0$

Momentum Theory: Forward Flight

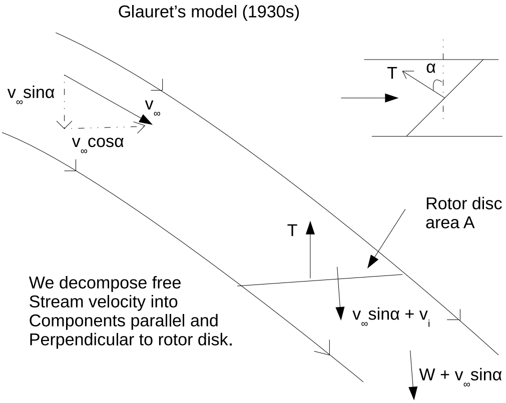


Figure: Glauret's flow model for the momentum analysis of a rotor in forward flight

Momentum Theory: Forward Flight

Advance ratio, $\mu = \frac{v_\infty \cos\alpha}{\Omega R}$
Inflow ratio,

$$\lambda = \frac{v_\infty \sin\alpha + v_i}{\Omega R} = \mu \frac{\sin\alpha}{\cos\alpha} + \lambda_i$$

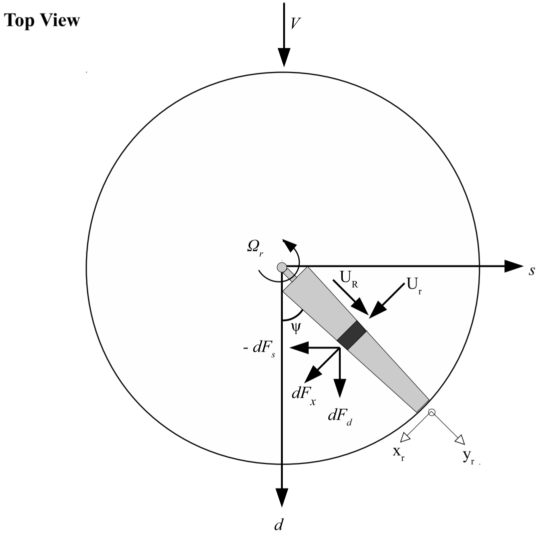
$$\lambda = \mu \tan\alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

This equation cannot be solved analytically but numerical solution is easy – Fixed point iteration or Newton Raphson can be used

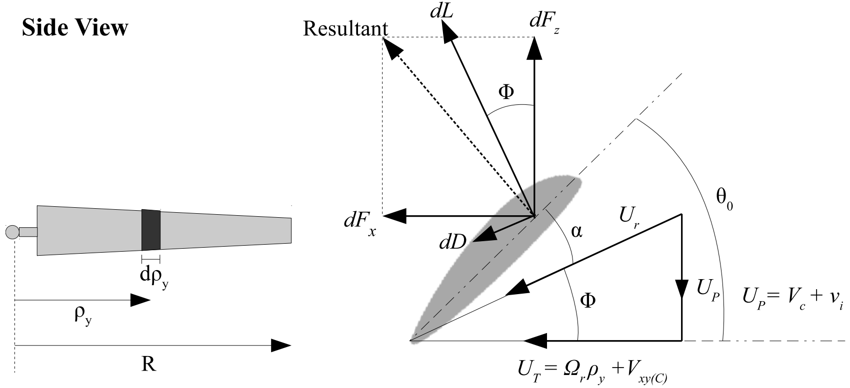
Subsection 2

Blade Element Theory

Rotor Blade Velocity Components



Blade Element



Aerodynamic Loads and Velocities

$$\left.\begin{aligned} dF_z &= dLcos(\Phi) - dDsin(\Phi) \\ dF_x &= dLsin(\Phi) + dDcos(\Phi) \\ dT &= N_b dF_z \\ dQ &= N_b y dF_x \end{aligned}\right\}$$

where

$$dL = (\frac{1}{2}\rho U_r^2 c C_l) d\rho_y \qquad U_r = \sqrt{U_P^2 + U_T^2}$$

$$dD = (\frac{1}{2}\rho U_r^2 c C_d) d\rho_y \qquad U_P = V_c + v_i$$

$$U_T = \Omega_r \rho_y + V_{xy(C)}$$

$$\alpha = \theta_0 - \Phi \qquad \Phi = \tan^{-1}\left(\frac{U_P}{U_T}\right)$$

Rotor Aerodynamics - 1

$$dT = N_b(dL\cos\phi - dD\sin\phi)$$
$$dQ = N_b y(dL\sin\phi + dD\cos\phi)$$

Applying small angle assumption for ϕ, $\phi = \frac{U_P}{U_T}$ $\quad U_r = U_T$
$dT = N_b dL$ and $dQ = N_b y(dL\phi + dD)$
Converting to non-dimensional quantities:
$r = \frac{y}{R}, \frac{u}{\Omega R} = \frac{u_T}{\Omega R} = \frac{\Omega y}{\Omega R} = \frac{y}{R} = r$
$dC_T = \frac{dT}{\rho A(\Omega R)^2} \qquad dC_Q = \frac{dQ}{\rho A(\Omega R)^2 R} \qquad dC_P = \frac{dP}{\rho A(\Omega R)^3}$

Rotor Aerodynamics - 2

$dC_T = \frac{dT}{\rho A(\Omega R)^2} = \frac{N_b dL}{\rho A(\Omega R)^2} = \frac{N_b(\frac{1}{2}\rho u_T^2 c C_l dy)}{\rho A(\Omega R)^2}$

$dC_T = \frac{N_b(\frac{1}{2}\rho(\Omega y)^2 c C_l dy)}{\rho \pi R^2(\Omega R)^2} = \frac{1}{2}(\frac{N_b c}{\pi R}) r^2 C_l dr$ where $\sigma = \frac{N_b c}{\pi R}$ is solidity

$$dC_T = \frac{1}{2}\sigma C_l r^2 dr$$

By similar approach, we find

$$dC_Q = \frac{1}{2}\sigma(\phi C_l + C_d) r^3 dr$$

where, ϕC_l is Induced drag and C_d is profile drag.

$$C_T = \frac{1}{2}\int_0^1 \sigma C_l r^2 dr$$

$$C_Q = \frac{1}{2}\int_0^1 (\phi C_l + C_d) r^3 dr = \frac{1}{2}\int_0^1 (\lambda C_l r^2 + C_d r^3) dr$$

Rotor Aerodynamics - 3

Assume $C_l = C_{l_\alpha} \cdot \alpha$

$$C_l = C_{l_\alpha}(\theta - \phi) = C_{l_\alpha}(\theta - \frac{\lambda}{r})$$

$$C_d = C_{d_o}$$

$$C_T = \frac{1}{2}\int_0^1 \sigma C_l r^2 dr = \frac{1}{2}\int_0^1 \sigma C_{l_\alpha}(\theta - \frac{\lambda}{r})r^2 dr$$

$$C_T = \frac{1}{2}\int_0^1 \sigma C_{l_\alpha}(\theta r^2 - \lambda r) dr$$

$$C_Q = \int_0^1 \lambda dC_T + \int_0^1 \frac{1}{2}\sigma C_d r^3 dr$$

Need a way to calculate inflow, uniform inflow from momentum theory can be used