Challenges in the modeling of metal forming processes

Uday Shanker Dixit
Department of mechanical Engineering
Indian Institute of Technology Guwahati
Processes that cause changes in the shape of solid metal parts via plastic (permanent) deformation are termed as metal forming processes.

In bulk metal forming processes, raw material and products have a relatively high ratio of volume to surface area.

**Introduction**

- **Rolling**
- **Extrusion**
Open–die forging

Closed–die forging
• **Wire drawing** is a process of pulling wires or rods through conical dies resulting in the reduction of its cross-section and increase in its length.

Wire drawing process
In sheet metal forming processes, raw material and products have a relatively low ratio of volume to surface area.

Deep drawing process. a Before deformation. b After deformation.
Schematic of laser forming process
Metal forming processes are also used just for improving the properties of the material.

**Severe Plastic Deformation Processes**

<table>
<thead>
<tr>
<th>Process name</th>
<th>Schematic representation</th>
<th>Equivalent plastic strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal channel angular extrusion (ECAE) (Segal, 1977)</td>
<td></td>
<td>( \varepsilon = n \frac{2}{\sqrt{3}} \cot(\varphi) )</td>
</tr>
<tr>
<td>High-pressure torsion (HPT) (Valiev et al., 1989)</td>
<td></td>
<td>( \varepsilon = \frac{n \pi r}{\sqrt{3}} ), ( \gamma(r) = n^2 \pi r )</td>
</tr>
<tr>
<td>Accumulative roll-bonding (ARB) (Saito, Tsuji, Utsunomiya, Sakai, 1998)</td>
<td></td>
<td>( \varepsilon = n \frac{2}{\sqrt{3}} \ln \left( \frac{a}{r} \right) )</td>
</tr>
</tbody>
</table>
Hydraulic Autofrettage

(a) During pressurization

(b) After the release of internal pressure
Purpose of modeling of metal forming processes

- The power requirement
- Pressure in the die and tools
- Stresses, strains and strain rate distributions in the material during processing
- Residual stress in the product
- Defects
- Geometric accuracy
- Surface integrity
- Mechanical and metallurgical properties
- Microstructure
Some of the methods for the analysis of metal forming processes:

1. Slab method:-
   - In this method, deformation of a workpiece can be approximated with the deformation of a series of slabs.
   - This method considers force equilibrium in the slabs.
   - Slab method has already been used in the analysis of various metal forming processes such as forging, rolling and extrusion.
   - Slab method has limited usefulness, but it is computationally very fast.
2. Slip line method:-

- This method is applied to the plane strain problem. Material is assumed to be rigid plastic.
- In the slip line method, the following equations are solved:

1. The yield criterion:

   For an isotropic material, the yield criterion may be written as

   \[ (\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = 4k^2 \]

   \[
   k = \begin{cases} 
   \frac{\sigma_y}{2} & \text{for Tresca criterion.} \\
   \frac{\sigma_y}{\sqrt{3}} & \text{for von Mises criterion.}
   \end{cases}
   \]
2. The equilibrium equations:

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0
\]

3. The continuity equations:

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0
\]
4. The rule of plastic flow for isotropic metals:-

The principal directions for the stresses and strain rate are same. Hence

\[(\sigma_x - \sigma_y)\left(\frac{\partial \nu_x}{\partial x} + \frac{\partial \nu_y}{\partial y}\right) = 2\tau_{xy}\left(\frac{\partial \nu_x}{\partial x} - \frac{\partial \nu_y}{\partial y}\right)\]

Unknowns \(\sigma_x, \sigma_y, \tau_{xy}, \nu_x, \nu_y\) (total 5 equations)
• It can be proved that characteristics lines for stress and velocity are in the direction of maximum shear stresses.

• Through each point in the plane of plastic flow, we may consider a pair of orthogonal curves along which the shear stress has its maximum value. These curves are called slip lines or shear lines.

(a) Stresses on the element  (b) Mohr’s circle
• Equations of Henky:-

\[ p + 2k\phi = \text{constant along an } \alpha \text{ line} \]
\[ p - 2k\phi = \text{constant along a } \beta \text{ line} \]

• Equations of Giringer:-

\[ du - vd\phi = 0 \quad \text{along an } \alpha \text{ line} \]
\[ dv + ud\phi = 0 \quad \text{along a } \beta \text{ line} \]

Thus, for a straight slip line, the hydrostatic pressure and the tangential velocity remains constant.
Slip line fields for the indentation of a semi-infinite medium by a punch

According to Henky’s equations, the punch pressure at yield point has the value

\[ q = 2k \left( 1 + \frac{\pi}{2} \right) \]
3. Upper bound method

• This method calculates the greatest possible load that a metal forming can deliver or sustain.
• The rate of work done by the unknown surface tractions is less than or equal to the rate of internal energy dissipated in any kinematic admissible field.

\[
\int_{S_u} t_i \, d\mathbf{u}_i \, dS_u \leq \int_{\nu} \sigma_{ij}^* \, d\varepsilon_{ij} \, d\nu + \sum_{S_D^*} \int_{S_D^*} k \, d\mathbf{u}^*_i \, dS_D^* - \int_{S_T^*} t_i \, d\mathbf{u}^*_i \, dS_T^*
\]
4. Lower bound method

- When a body is yielding and small incremental displacements are undergone, the increment of work done by the actual forces or surface tractions on $S_u$ is greater than or equal to that done by the surface tractions of any other statically admissible stress field,

$$\int_{S_u} T_i \, d\mathbf{u}_i \, dS_u \geq \int_{S_u} T_i^* \, d\mathbf{u}_i \, dS$$

- This method has less popularity in metal forming than upper bound method.
5. Visioplasticity

- This method combines experiment and analysis.
- A velocity field is obtained from a series of photographs of the instantaneous grid pattern during actual forming process.
- Strain rate, stress and strain fields can be obtained from the considerations of the equilibrium and constitutive equations.

The method has limited applications.
Governing equations for FEM models

- Continuity equation \( \dot{\varepsilon}_{ii} = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = 0 \)
- Equilibrium equations

\[
\sigma_{ij,j} + b_i = -p_i + S_{ij,j} + b_i = 0
\]

- Strain-rate-velocity relations
- Incremental strain-displacement relations
- Constitutive relations
FEM Technique

• Finite element analysis:
  • FEM is the most preferred technique, as it can easily include non-homogeneity of deformation, process dependent material properties and different friction models.

1. Lagrangian formulation:
  • Lagrangian formulation represents a more natural and effective approach than an Eulerian approach for metal forming.
  • In an Eulerian formulation of a structural problem with large displacements, new control volumes have to be created (because the boundaries of the solid change continuously) and the non-linearities in the convective acceleration terms are difficult to deal with.
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1. Lagrangian formulation:

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• In an Eulerian formulation of a structural problem with large displacements, new control volumes have to be created (because the boundaries of the solid change continuously) and the non-linearities in the convective acceleration terms are difficult to deal with.
For motion of material particle $P$, in a two dimensional coordinate system, the relation between the spatial position ($x$) and the initial coordinates ($X$) and time ($t$) is given by

$$x = x(X, t)$$

The above equation expresses a material description of motion in a Lagrangian formulation.

The initial configuration at $X$ provides a reference configuration to which all future variables are referred.
Updated Lagrangian formulation:

- For large deformation problems, the Lagrangian formulation proves to be cumbersome with the governing equations being difficult to solve.
- In updated Lagrangian formulation, it is assumed that the states of stress and deformation of the body are known till the current configuration, say time $\tau$.
- The main objective is to determine the incremental deformation and stresses during the time step $\Delta t$ and the state of the system in a future configuration, at time $\tau+\Delta t$.
- In UL formulation
  \[ x = x(x(\tau), \tau+\Delta t) \]
• **Difficulties in updated Lagrangian formulation:**
  
  - More computational time.
  
  - Some times, residual stresses are not predicted properly, due to accumulation of computational errors.
  
  - Difficulty in applying spatial boundary condition.
Eulerian formulation:

- This method is found to be suitable for the rigid-plastic analysis of steady-state processes.
- In Eulerian formulation, a spatial description is used, whose independent variables are present position $x$ occupied by the particle and the present time $t$.
- Formulation of metal forming problems using this approach is called flow formulation.
- In flow formulation, the primary unknown variables from finite element analysis are the velocities/ hydrostatic pressure at different locations i.e. nodes.
The following relations exists between strain rates and the velocity gradient:

\[ \dot{\varepsilon}_{ij} = \frac{1}{2} \left( v_{i,j} + v_{j,i} \right) \]

where \( v_{i,j} \) is the partial derivative of the \( i^{th} \) component of velocity with respect to \( j^{th} \) component of position vector.

Levi-Mises plastic flow rule is given by,

\[ S_{ij} = 2\mu \dot{\varepsilon}_{ij} \]

\( \mu \) is the Levi-Mises coefficient,

\[ \mu = \frac{\sigma_y}{3 \tilde{\varepsilon}} \]
\( \tilde{\varepsilon} \) is equivalent strain which is equal to \( \sqrt\frac{2}{3} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij} \)

\( (\sigma_y)_0 \) is the flow stress at zero strain while \( b, n \) are determined from experiments. Neglecting the effect of strain rate and temperature, flow stress can be expressed as

\[
\sigma_y = (\sigma_y)_0 \left(1 + \frac{\tilde{\varepsilon}}{b}\right)^n
\]
FEM Technique

- The continuity and equilibrium equations:
  \[ v_{i,i} = 0 \]
  \[-p_{,j} + S_{ij,j} = 0 \]

  where \( p \) is the hydrostatic pressure.

- These equations are similar to Navier-Stokes equations.

- Many researchers attempted to solve these equations for different metal forming processes using Galerkin (weak) formulation.

- Absence of pressure term in the continuity equation makes the above method difficult.
• The pressure may be included in the continuity equation by writing it as

\[ \nu_{i,i} + \frac{p}{\lambda} = 0 \]

where \( \lambda \) is a penalty parameter.

**Direct Penalty Method**

• High value of \( \lambda \) makes the system of equations ill-conditioned. Low values introduce approximations.

**Mixed Pressure-velocity formulation**

• House-holder method can be incorporated to handle ill-conditioned system of equations.
• The flow formulation has been extensively used for the analysis of wire drawing, extrusion, rolling etc.

Example of a cold flat rolling analysis using flow formulation.

Discretised metal strip for FEM analysis [Dixit and Dixit, 1996]
• In the FEM analysis of cold rolling [Dixit and Dixit, 1996], Galerkin method has been used and the global equations after applying the boundary conditions, are in the form of:

\[ [K]\{\Delta\} = \{F\} \]

where

\([K]\) is the global coefficient matrix,
\(\{F\}\) is the global right hand side vector,
\(\{\Delta\}\) is the global vector of nodal values of pressure and velocity (primary variables).

• The solution has been obtained in the form of nodal velocities and pressure, then the secondary variables like roll force and roll torque can be computed.
Formulation may not require any pressure boundary conditions.
The pressure values may be determined with an additive constant. The constant can be eliminated from traction conditions.
In the direct penalty method:

\[ p = -\lambda \times \dot{\varepsilon}_{ii} = -\infty \times 0 = \text{indeterminate} \]
Pressure computation by FDM

The domain showing plastic boundaries and the points for finite difference approximation

Comparison of FEM and experimental results for roll force and roll torque (Steel, $R/h_1 = 65$)

Comparison of FEM and experimental results for roll force and roll torque (Steel, $R/h_1 = 130$)

Comparison of FEM and experimental results for roll force and roll torque (Copper)

Comparison of roll pressure distribution for different methods

Some points regarding estimation of secondary quantities

- Finding the load of deformation: Integration of stresses versus Energy balance
- Calculation of normal stresses and shear stresses at the tool–work interface

\[ t_s = f t_n \]

Should I find \( t_s \) by above equation or directly from stress components?
Comparison with Wistreich’s results of wire drawing

Numerical aspects are very important

\[ R = 65 \text{ mm}, \% r = 16, \frac{R}{h_1} = 65 \]

- **Roll torque/unit width (kN.m/m)**

- **f**

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Central burst in wire drawing: high die angle, low reduction

In the rigid plastic finite element analysis of temper rolling, [Chandra and Dixit, 2004], showed that the non-homogeneous deformation causes region of very low strain-rate (less than 3% of maximum strain-rate), indicating the presence of elastic zone in between the plastic zone.

Equivalent strain-rate contours for \( r = 4 \% \), \( \mu = 0.3 \), \( R/h_1 = 130 \)
Variation of tangential stress with normal stress according to Wanheim and Bay friction model.
In the finite element analysis of flat rolling with inclusion of anisotropy [Dixit and Dixit, 1996], it has been shown that values of equivalent strain-rate and equivalent strain decrease with $m$ which is a parameter that governs the shape of the yield surface.

$\begin{align*}
\text{(a)} & \quad m = 1.5 \\
\text{(b)} & \quad m = 2
\end{align*}$

Equivalent strain-rate contours for two different cases of $m$
• Analysis of residual stresses, is a difficult area in the metal forming process.

• Dixit and Dixit, (1995) proposed, a simplified approach to find the longitudinal residual stress (stress in the direction of rolling) which may prove, an economical analytical method.

• Three different approaches *i.e.* mixed formulation, method of multiplicative decomposition of the deformation and the rate formulation were discussed for analyzing elasto-plastic rolling problem.
Thermal Modelling: warm flat rolling

Constitutive relation
- Strip temperature
- Roll temperature
- Average temperature of roll and strip at the interface
- Heat partition factor $\lambda$
- Roll-force, Roll-torque, Distributions of stress, strain, strain-rate
- Slip

Thermal module
- Based on Eulerian flow formulation
- Based on Analytical methods and/or
- Based on FEM using ABAQUS

Deformation module
- Modelling of warm flat rolling
- Thermal module

An overview of thermo-mechanical model of plane strain rolling

Schematic of upper half view of the roll and strip
Thermal Autofrettage

A schematic diagram of thermal autofrettage process, (b) Elastic-plastic zones across the wall thickness
<table>
<thead>
<tr>
<th>S. No.</th>
<th>Empirical model</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\sigma = K\varepsilon^n$</td>
<td>Hollomon law, deviates at low strain, for high strain $\varepsilon$ may be treated total as well as plastic strain rate but for low strain it refers to plastic strain</td>
</tr>
<tr>
<td>2.</td>
<td>$\sigma = \sigma_y + K\varepsilon^n$</td>
<td>Ludwik law, $\varepsilon$ is plastic strain, does not give good fit over a wide range</td>
</tr>
</tbody>
</table>
| 3.    | $\sigma = \sigma_y(1 + \varepsilon/b)^n$  
$\sigma = C(m + \varepsilon)^n$ | Swift’s generalized power law, suitable for a wide range, $\varepsilon$ should be plastic strain when elastic and plastic strains are of comparable magnitude |
| 4.    | $\sigma = \sigma_y + K[1 - me^{-n\varepsilon}]$ | Voce law, $\varepsilon$ should be plastic strain when elastic and plastic strains are of comparable magnitude |
| 5.    | $\varepsilon = \frac{\sigma}{E} \left[ 1 + \alpha \left( \frac{\sigma}{\sigma_0} \right)^{m-1} \right]$ | Ramburg-Osgood equation, considers elasticity, $\varepsilon$ is total strain |
| 6.    | $\sigma = \sigma_y \tanh \left( \frac{E\varepsilon}{\sigma_y} \right)$ | Pragar’s law for ideally plastic material, $\varepsilon$ is total strain |
| 7.    | $Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) = A \{\sinh (\alpha \sigma)\}^n$ | Relation considering strain-rate and temperature, $Z$ is Hollomon parameter or temperature corrected (plastic) strain-rate |
| 8.    | $\sigma = k\varepsilon^n \dot{\varepsilon}^m \exp \left( \frac{\beta}{T} \right)$ | Relation considering strain, strain-rate and temperature, $m$ is (plastic) strain-rate sensitivity |
| 9.    | $\sigma = (A + B\varepsilon^n) \left( 1 + C \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right) \left( 1 - \left( \frac{T - T_0}{T_{melt} - T_0} \right)^m \right)$ | Johnson-Cook Model, widely used in machining, strain and strain-rate are usually taken plastic |
| 10.   | $\sigma = \sigma_0 \varepsilon^n \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m \left( \frac{T}{T_0} \right)^{-r}$ | Power law |
Assessing the flatness of rolled sheet
The strip curvature due to difference in axial strains.

Strip curvature due to difference in shear strains.
Curvature index, $K$

$$K = \frac{R - h_2/2}{R + h_2/2},$$

$R_n = R_i = 96.9\,\text{mm}$

$m_a = m_l = 1$

$h_i = 63.5\,\text{mm}$

$r = 25\%$

Experimental [22]
Experimental [23]
FEM [13]
Present model
Radius of curvature (m)

$V_A = 1.02$
$V_A = 1.04$
$V_A = 1.08$

$R_u = R_l = 350$ mm, $r = 20\%$

$\mu_u = \mu_l = 0.12$, $Y_s = 169.9$ MPa

$b = 0.05$, $n = 0.26$

Input thickness (mm)
Effect of coefficient of friction on the radius of curvature.

- $h_i = 3 \text{ mm}$
- $h_i = 6 \text{ mm}$
- $h_i = 9 \text{ mm}$

$R_u = R_i = 350 \text{ mm}$

$\mu_u = \mu_i = 0.12$, $V_A = 1.05$

$r = 20\%$, $Y_z = 169.9 \text{ MPa}$

$b = 0.05$, $n = 0.26$
Effect of $V_A$ on the radius of curvature for different reductions.

- $r = 10\%$
- $r = 20\%$
- $r = 30\%$

$R_u = R_l = 350\, \text{mm}$

$\mu_u = \mu_l = 0.12, r = 20\%$

$Y_s = 169.9\, \text{MPa}$

$b = 0.05, n = 0.26$
Radius of curvature (m)

- $h_i = 3 \text{ mm}$
- $h_i = 6 \text{ mm}$
- $h_i = 9 \text{ mm}$

$R_u = R_l = 350 \text{ mm}$

$\mu_u = \mu_l = 0.12, V_A = 1.05$

$Y_s = 169.9 \text{ MPa}$
The cold metal rolling industry needs reliable and accurate models to improve predictions of surface finish and friction and thus increase productivity and improve quality.

State of affairs under mixed lubrication [Xie et al. [2011]]

Crystal plasticity finite element modelling of surface roughness in rolling process [Jiang et al. 2013]

- With an increase of reduction, the surface roughness of workpiece decreases significantly.
- Lubrication can delay the process of surface asperity flattening.
- Increasing strain rate can lead to a decrease of surface roughness under the same reduction.

Effect of work roll on surface roughness at rolling temperature 850 °C

Effect of work roll on surface roughness at rolling temperature 950 °C

Crystal plasticity finite element modelling of surface roughness in rolling process [Jiang et al. 2013]

Relation between the surface roughness and friction/reduction (a) with lubrication and (b) without lubrication

Experimental study (Yadav, 2016)

- The friction and surface roughness effects on edge crack evolution of thin strip rolling.

- For improving the quality of rolled sheet, the surface roughness and friction are the most important controllable parameters.

- Surface roughness of the rolled strip was measured using the surface roughness measuring instrument (Pocket Surf).

Averaged measured surface roughness of the strip after rolling at different reductions for \( h_1 = 5 \) mm, \( R = 100 \) mm (Values in bracket are standard deviations)

<table>
<thead>
<tr>
<th>Reduction (%)</th>
<th>Measured surface roughness after cold rolling ( R_a (\mu m) )</th>
<th>Measured surface roughness after warm rolling ( R_a (\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rolling direction</td>
<td>Transverse direction</td>
</tr>
<tr>
<td>8.2</td>
<td>0.411 (0.026)</td>
<td>0.379 (0.014)</td>
</tr>
<tr>
<td>12.4</td>
<td>0.368 (0.024)</td>
<td>0.349 (0.025)</td>
</tr>
<tr>
<td>22.2</td>
<td>0.336 (0.033)</td>
<td>0.299 (0.018)</td>
</tr>
<tr>
<td>32.6</td>
<td>0.288 (0.012)</td>
<td>0.281 (0.021)</td>
</tr>
</tbody>
</table>
Modelling of foil rolling
Inverse Modelling for estimation of parameters

A laboratory rolling mill (a) front view and (b) arrangement for measuring the temperature and velocity of exit strip at the rear side.
Experimental validation of strategy for inverse estimation

\[ \sigma_f = \sigma_0 e^{n \left( \frac{T}{T_m} \right)^\gamma}. \]

The following objective function is to be minimized with respect to the decision variables \( \sigma_0, n, \) and \( \mu \) such as

\[ E = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{T_{ism} - T_{isc}}{T_{ism}} \right)^2} \]

Flow chart of the inverse model for mechanical properties and friction coefficient determination
Comparison of the experimental stress–strain curves and inversely estimated stress–strain curves at different temperatures with upper and lower bound fit data (a) Room temperature, (b) 100 °C, (c) 150 °C and (d) 250 °C
**Meshless Methods**

- **Meshless methods:**

  - Group of numerical methods for solving partial differential equations on regular or irregular distribution of points.
  - Costly mesh generation and remeshing is not required.
  - Some methods are based on weak form of differential equations, whilst others use collocation points.
  - These methods lack in robustness and computational speed, even though in some cases, the quality of solution has been shown to be better than that obtained by FEM.
  - These methods have not been extensively applied for solving the problems in metal forming industry.
Reproducing kernel particle method (RKPM):
Chen et al. have used RKPM for the analysis of ring compression and upsetting problem.
Shangwu et al. utilized RKPM method for modeling of plane strain rolling problem.

Corrected smooth particle hydrodynamics (CSPH):
Bonnet and Kulasegaram et al. utilized the CSPH to perform two-dimensional simulations of several metal forming processes without considering strain-hardening.

Element free Galerkin method:
Xiang et al. applied element free Galerkin method for simulation of plane strain rolling.
• **Radial basis function collocation method:**
  - An axisymmetric forging problem has been analyzed using this method.
  - In this method, Boundary conditions and governing equations are satisfied at the collocation points.

\[
\phi = \sum w_i g_i \left( \| x - x_k \|, c_k \right)
\]

- **Multiquadric**
  \[
g = \sqrt{r^2 + c_k^2}
\]
- **Gaussian**
  \[
g = \exp \left( -\frac{r^2}{c_k^2} \right)
\]
- **Inverse multiquadric**
  \[
g = \frac{1}{\sqrt{r^2 + c_k^2}}
\]
- **Thin-plate splines**
  \[
g = r^{2m} \ln r
\]
Simulation of the process using RBF collocation method

• A typical example of an axisymmetric forging problem:

Meshless Methods

Problem domain
• Typical contours:

Non-dimensional equivalent strain rate at 20% reduction in height with $f = 0.05$ and $R/H = 1/2$.

Equivalent plastic strain at 20% reduction in height with $f = 0.05$ and $R/H = 1/2$. 
Contours of hydrostatic pressure at 20% reduction in height with $f = 0.05$ and $R/H = 1/2$

Contours of normal stress at 20% reduction in height with $f = 0.05$ and $R/H = 1/2$
Computational Efficiency: Hybrid Methods

A general overview of the NN-assisted FEM model.
Further Research Directions

• Research directions requiring special attention:
  • The field of modeling of metal forming process is far away from saturation.
  • The figure shows some of the areas in which research is needed.
The constitutive behavior of isotropic and anisotropic material has to be understood.

The expressions for determination of flow stress have to be developed.

In metal forming, the understanding of the physics of the problem is of utmost important for developing a suitable computational techniques.

Computational techniques like Updated Lagrangian method requires a lot of memory and computational time.

Implementation of adaptive mesh refinement method will speed up the computation of the FEM techniques.
Further Research Directions

- Tribological aspect is also a major area to be explored in the modeling of metal forming.

- Use of constant coefficient of friction may give different results in the case of foil rolling process.

- There seems to be no point in refining the rolling models unless more is known about the nature of friction in the roll bite. [Fleck et al., 1992].

- Kumar and Dixit (2005) incorporated a more realistic friction model, i.e. Wanheim and Bay’s model for cold foil rolling and observed that friction model has great influence on qualitative and quantitative predictions of foil rolling processes.
Further Research Directions

- The prediction of microstructure is also a lesser explored area.

- The micro-structure evolution occurs by recovery, recrystallization and grain growth phenomenon.

- Finite element in conjunction with microstructure modeling is expected to provide better judgment for the analysis of the defects.

- The blending of macro and micro modeling will provide useful results for industries.
Further Research Directions

• Analysis of surface roughness is one area where lot of experimental as well as computational work has to be carried out.

• More efforts are needed to explore this field for the prediction of the surface roughness including the type of lubrication used to carry out the desired metal forming process.

• Analysis of surface roughness can be useful in exploring the various friction models as friction plays a major role in determining the surface roughness.